Connection and Disconnection of Networks

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Abstract

When two networks interconnect, one network may value the connection more than the other. In an unregulated environment, the direction of payment for interconnection depends on whether there are increasing or decreasing marginal returns to network size. If users face decreasing marginal returns to network size, a small network benefits more from interconnection than a large network. Assuming that pricing between networks reflects this asymmetry, large networks can charge smaller ones for interconnection. Network mergers can result in higher interconnection fees for non-merging networks. Conditions for disconnection are examined. Despite the increasing value of network size, a large network may disconnect, either to recruit members of the disconnected network or, surprisingly, to shrink aggregate network size. Shrinkage benefits the large network in bargaining with surviving networks.

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1 Introduction

In a point-to-point network, a member joins the network of one particular firm and values that membership based on the number of other individuals, or points, the member can reach. Because some individuals belong to other networks, members of a point-to-point network typically value connections between their network and other networks.[1] By connecting to others, a network increases the quality of the product it offers to its members and can increase the price it charges its members. Once connected to other networks, the network serves both as a supplier to all its connected networks, by providing other networks with access to its own members, and as a buyer, or receiver, from all its connected networks, by gaining access to the members of connected networks. Point-to-point networks exist in a variety of industries, including telecommunications, energy, banking, and transportation.

Frequently, the terms of connections between individual networks are regulated. The nature of the regulation varies, however, and the reasons for the variation are not always apparent. Most importantly, unless unregulated incentives yield market distortions, there is little economic rationale for the regulation of interconnection prevalent in some network industries. In this paper we seek to understand what sorts of distortions, if any, arise from the incentives governing network interconnection.

We begin with a basic question: Why do firms connect their networks to each other? The benefits to individual members of increased

[1] There are at least two exceptions to this generalization: (1) when all the people with whom one values communicating are already on one's network, as might happen for a company's worker who only wants to send and receive emails to other people at that company or (2) when other networks provide more negative effects, such as from junk email, than positive effects.

aggregate network size certainly play a role in explaining why networks interconnect. However, the precise incentive mechanism that governs interconnection decisions of firms is relatively poorly understood. Improving our understanding of these incentives is important for deciding both whether and how to regulate network industries. This paper develops a model of how incentives operate and examines the implications of that model. The foundation for the analysis is the derivation of an individual network's incentives from aggregating the representative member's value of network participation over the network's members. Narrowly, our goal is to explain the behavior of Internet backbone networks. More broadly, though, the goal is to explain how the key characteristics of interconnection behavior arise from individual utility functions.

We assume that all potential members are already members of a network, in contrast with the model of Crémer, Rey, and Tirole (2000) in which new subscribers are joining networks and the expectations of new subscribers drive the ultimate network structure (see Katz and Shapiro (1985).) We thus focus on mature networks as opposed to growing networks. Understanding the behavior of mature networks is important since many networks are relatively mature, including the Internet.[2] When networks have reached a mature stage and there are switching costs, the subscribers' expectations are less important to a network's choices since the subscribers already experience lock-in effects. The primary choices faced by mature networks govern interconnection fees and whether to interconnect at all. These choices are regularly faced by Internet providers.

[2]Focusing on the Internet as a mature market is reasonable because growth of new subscribers has fallen dramatically in recent years. AOL's revenue from members grew more than 50% between 1998 and 1999 America Online (2000), but that growth had already fallen to less than 15% between 2000 and 2001 AOL Time Warner (2001).

The Internet is an aggregate network consisting of many individual, unregulated networks. Each Internet provider could exist independently and serve its customers independently.[3] However, most service providers have chosen to connect with each other, either directly or via intermediaries, even in the absence of regulations that require them to do so.[4] Because of these connections, a user of a particular network can generally contact a user of another network without significant difficulty. Nonetheless, two networks may value their connection differently even if they send the same quantity of traffic in each direction across their interface. A critical question then becomes what terms of connection will be set.

Interestingly, the carriers of Internet traffic with the largest networks generally do not charge each other to exchange traffic.[5] However, the large networks do charge smaller networks for connectivity.[6] Thus there is an asymmetry in the pricing of interconnection that is related to the size of networks. We develop a model that explains such asymmetric pricing even in the absence of cost differences.

To understand the incentives for interconnection between networks, we must first understand how a network earns profits. The revenue value of a network is the sum of its members' network values.

[3] In this paper, an Internet provider is a backbone provider, a long-distance Internet company, capable of picking up and delivering Internet protocol messages from multiple locations in the US and world. Internet backbone providers are assumed to have similar costs per member and similar technical capabilities.

[4] Networks that did not provide full web interconnection in the past, such as AOL and Prodigy, now provide full web interconnection. Note, though, that even now, the Internet does not provide truly universal interconnection. There are still some networks, typically in developing countries, that cannot be reached from others.

[5] These carriers are frequently called Tier 1 Internet Backbone Providers, but this paper refrains from using this terminology, instead focusing on more externally identifiable features such as the number of subscribers. This paper shows that the contractual nexus of Tier 1 providers and the free exchange of traffic between them may be seen as a consequence of the externally identifiable features.

[6] See §27 and §54 of European Commission (2000).

The primary factor in the calculus of interconnection is then the extra value each network derives from connecting to the other, based on each member receiving an incremental increase in the value of belonging to an enhanced network. When one network benefits more from interconnection than the other, a bargaining outcome arises in which the surplus from interconnection is shared. The network that receives the most value from interconnection pays the network that receives less value.

In particular, we show that the benefits of bilateral interconnection for two networks of different sizes vary systematically depending on the marginal value to a user of adding new users to a network. Generally, when consumers experience decreasing marginal value from adding new users to their network, the benefit of interconnection is likely to be greater for the smaller network than for the larger network.[7] In this situation, one might expect the small network to pay the large network for interconnection. Chipty and Snyder (1999) develop a simpler one-sided version of this intuition applied to the bargaining between a unique supplier and multiple buyers in the context of cable programming. The current model is distinct because, in point-to-point networks, multiple buyers are suppliers to each other and because the behavior of networks in this model is derived from individual value functions.

Conversely, when consumers experience increasing marginal value from adding new users to their network, the total benefit to a large network of interconnection to a small network is greater than the value to the small network. In such a situation, one might expect the large network to pay the small network for interconnection.

[7] This intuition has been mentioned by Milgrom, Mitchell, and Sringesh (1999), but has not yet been formalized or systematically analyzed to my knowledge.

Initially, we focus on equilibria in which networks threaten disconnection but ultimately connect. Full interconnection is a desirable outcome because the social value of a set of networks is generally largest with full interconnection. Yet there are conditions under which one network does disconnect from another.

Two types of disconnection can occur. The first arises when the large network expects to recruit the members of the disconnected network, leaving the aggregate network size unchanged. The second is more surprising: Disconnection may also occur when the members of the smaller disconnected network exit and both the aggregate network size and the social value shrink. This is because a large firm's bargaining position with the remaining networks may be strengthened when the aggregate network is smaller.

This paper models the value of interconnection, demonstrates how mergers may impact the incentives for bilateral interconnection, derives the incentives to disconnect networks, and considers the welfare impacts of mergers and disconnection.

2 Model

The model focuses on the different values of bilateral interconnection between networks and how those values vary with asymmetric network sizes. We assume that two networks send identical quantities of information in each direction across their network interface.[8] Thus any difference in the value of interconnection that we may find is not the result of asymmetric inbound and outbound traffic but is instead

[8]This pattern of traffic could arise in the context of email, when sending an email results in a return email, or in video conferencing, when each person is sending a large quantity of audiovisual information while simultaneously receiving that amount of information. Clearly, in these contexts, two networks of very different sizes may have identical traffic flow in each direction.

the result of different network sizes.

In this model, no regulations require interconnection or set any terms for interconnection. In particular, there is no requirement that interconnection charges be symmetric. This contrasts with other recent work such as Armstrong (1998) and Laffont, Marcus, Rey, and Tirole (2001) where access charges are set symmetrically. When access charges are set symmetrically, two networks sending equal amounts of traffic to each other will not make net payments to each other. In general, there is no reason to believe this restriction would apply in absence of regulation. It is important to relax the symmetry requirement to analyze the asymmetric incentives for interconnection.

A network is allowed to charge another network a fee for interconnection as well as for traffic carried. The net result of these charges is the interconnection fee.[9] Contracts are bilateral and independent. Each network has the option of not interconnecting at all. As long as interconnection generates more value to a network's users, the network will choose to interconnect.

A user of a network derives a value $v(n)$ from belonging to a network with n users. $v(n)$ is twice continuously differentiable. Generally, as network size increases, the value to the user increases. That is, $v'(n) > 0$.[10] The stronger the network effects, the greater is $v'(n)$. If network i has n_i users, then the non-interconnected network value is the sum of the individual values, or $n_i v(n_i)$. On all τ networks jointly,

[9]Such a net price could be achieved, for example, by two networks setting fixed charges to each other for interconnecting their two networks or by setting different per unit rates for traffic going in one direction or another.

[10]We could define $v(n)$ as $v(n) = u(n) - c(n)$ where $u(n)$ represents the value provided by network size, while $c(n)$ represents the user costs of having more users on the network, possibly from increased congestion. Generally, then, $v'(n)$ may be greater than or less than 0, depending on whether the congestion costs outweigh the network size benefits.

there are a total of N users, where

$$N = \sum_{i=1}^{\tau} n_i.$$

If all networks connect with each other, then an individual derives benefit from connecting with $N - 1$ other users. The maximum value to a user of joining a network is thus $v(N)$. The minimum value of network membership is 0, since a network with just one member yields no network benefits, or $v(1) = 0$.

Each user is a member of one and only one network. Members have a switching cost s that is the sum of a network exit cost s_e and a network joining cost s_j.[11] For simplicity, we assume switching costs are such that $s_e + s_j > v(N)$. Users will thus not choose to switch networks. If a member's network ceases operation, the member will necessarily experience the exit cost so s_e will not be a choice variable. The member may then subscribe to another network, paying just an entry portion of the switching costs s_j. When networks are interconnected, a member will only resubscribe for $s_j < v(N)$. We examine a mature network structure in which all members have already incurred a joining cost in the past. Networks charge existing subscribers the full current value of the network. If the value of the network to a user rises or falls, the charge to the user will rise or fall commensurately.

Physical costs of network operation are c per subscribing member.[12] For analytical ease, there is no cost of interconnection. We assume that $v(n_i) > c, \forall i$ unless stated otherwise.

Suppose there are two networks, network i and network j, that are considering interconnecting and that have the technical capability

[11] These switching costs might arise, for instance, from a user's fixed email address that cannot be ported to another network.

[12] Note that it is beyond the scope of the current analysis to account for the potential increased costs of network operation when there is increased traffic.

to connect because they have compatible technology. They have n_i members and n_j members respectively. How does each network value the interconnection?

The value of interconnection to a network can be represented as the difference between its network value when it connects and its network value when it does not connect. This measures the opportunity cost of not connecting.

When connected, the members of network i thus derive a value of interconnection with network j of

$$V_i = n_i v(n_i + n_j) - n_i v(n_i). \tag{1}$$

Similarly, network j derives a value of interconnection with network i of

$$V_j = n_j v(n_j + n_i) - n_j v(n_j). \tag{2}$$

Even though networks i and j will exchange equivalent quantities of traffic with each other, V_i and V_j need not be equivalent. When there is a difference between the values, the network with the greater value from interconnection will then pay the other network for the interconnection, as a result of negotiation over mutually beneficial gains based on the bilateral Nash bargaining solution (Nash (1950) and Lopomo and Ok (2001).)[13] The economic opportunity for one network to charge another for interconnection arises since, after interconnection, each network experiences increased revenues from the extraction of increased values from members. In this paper, we focus on the difference in the values of interconnection that might drive bargaining over bilateral interconnection contracts.

[13]Note that networks do behave in ways consistent with a Nash bargaining solution. For example, the physical costs of interconnection are split evenly between large networks. See §24 of US Department of Justice (2000).

3 Interconnection

We begin by comparing the values of interconnection between two networks. We consider the case in which two networks are of the same size, the case in which the network value function is linear, and then the cases in which the network value function is concave and convex.

Proposition 1 *Let network i and network j be two potential interconnecting networks, with n_i members and n_j members respectively.*
(i) When $n_i = n_j$, each network receives an equal value from interconnection.
(ii) If $v'(n) = l$, l a constant, then the networks receive an equal value from interconnection.
(iii) Let $n_j > n_i$. If $v'(n) > 0$, for $n > 0$, then
(a) when $v''(n) < 0$, network i derives a greater value from interconnection than network j.
(b) when $v''(n) > 0$, network j derives a greater value from interconnection than network i.

Proof. **(i)** $V_i = V_j$ trivially when $n_i = n_j$.
(ii) $V_i = n_i[v(n_i + n_j) - v(n_i)]$. Similarly, $V_j = n_j[v(n_j + n_i) - v(n_j)]$. Because $v'(n) = l$, $v(n_i + n_j) - v(n_i) = n_j l$. Similarly, $v(n_j + n_i) - v(n_j) = n_i l$. As a result, $V_i = n_i n_j l$ and $V_j = n_j n_i l$, so $V_i = V_j$.
(iii)(a). We want to show that the value of interconnection for the small network i (from equation (1)) is greater than the value of interconnection for the larger network j (from equation (2)) when the value function is concave, or

$$n_i v(n_i + n_j) - n_i v(n_i) > n_j v(n_i + n_j) - n_j v(n_j).$$

By concavity, we know that that the value function is located above the straight line that connects the points $(n_i, v(n_i))$ and $(N, v(N))$. In particular, $v(n_j)$ is located above the linear combination at n_j. That is,

$$v(n_j) > L(n_j) \tag{3}$$

for $n_j \in (n_i, N)$, where $L(n_j)$ is the linear combination of $(n_i, v(n_i))$ and $(N, v(N))$ at the value n_j. The linear combination $L(n_j)$ is given by

$$L(n_j) = \frac{[(n_i + n_j) - (n_j)]v(n_i) + (n_j - n_i)v(n_i + n_j)}{(n_i + n_j) - (n_i)}.$$

The expression for $L(n_j)$ simplifies to

$$L(n_j) = \frac{n_i v(n_i) + (n_j - n_i)v(n_i + n_j)}{n_j}. \tag{4}$$

Substituting equation (4) into the inequality (3),

$$v(n_j) > \frac{n_i v(n_i) + (n_j - n_i)v(n_i + n_j)}{n_j}$$

and rearranging,

$$n_i v(n_i + n_j) - n_i v(n_i) > n_j v(n_i + n_j) - n_j v(n_j)$$

or $V_i > V_j$. $\square$

(iii)(b). Comparable to (i), using definition of convexity. $\square$

Proposition 1(i) states that the value of interconnecting and the value of not interconnecting are the same for each network. As a result, neither benefits more than the other from interconnection.[14]

The Nash bargaining solution suggests that when there are no value differences, we might expect no interconnection fee. Such a result would be consistent with contracting practices between Internet providers; similar-sized providers typically do not charge each other for interconnection.[15]

[14]No claim is made for the originality of propositions 1(i) and 1(ii), though I am aware of no work that reports them.

[15]See §27 of US Department of Justice (2000).

When two networks have different numbers of users, the difference in the interconnection values depends critically on the properties of the function $v(n)$, as shown in Propositions 1(ii) and 1(iii). First consider the situation in which two networks have a different number of members but the representative member has a linear network value function.

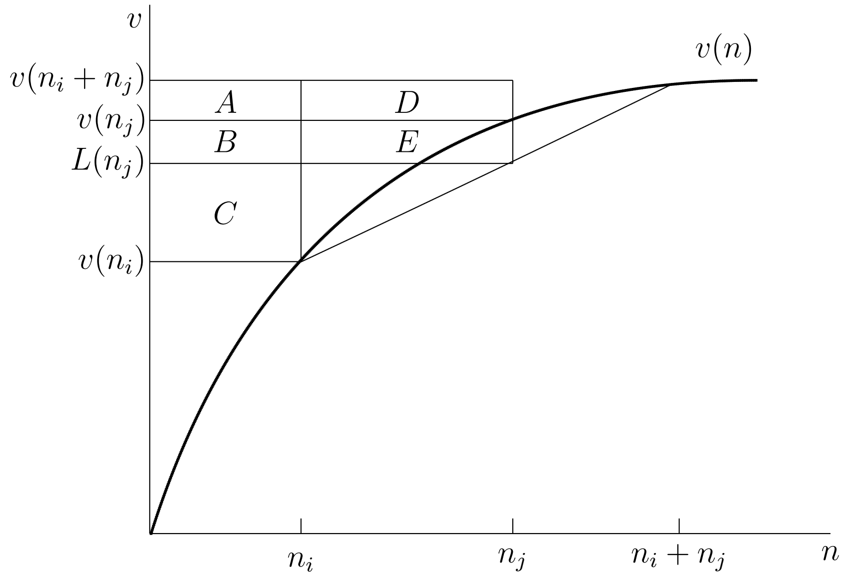
When $v(n)$ is linear, the value of interconnection will be mutually symmetric, even between a large network and a small network as shown in proposition 1(ii). A user of a small network will value interconnection more than the user of a large network by an amount that is directly proportional to the size of the two networks. However, the difference in values will be precisely counterweighted by the difference in the number of members of each network. As a result, the networks' value of interconnection will be identical.

In contrast, when the member's value function is non-linear, we find that large and small networks place different values on interconnection. Depending on the concavity or convexity of the network value function, the larger network will sometimes value interconnection more than the smaller one and the smaller one will sometimes value interconnection more than the larger one.

To provide a graphical intuition for the concavity result of Proposition 1(iiia), consider Figure 1. We know that n_j lies between n_i and N. Moreover, the individual's network value curve lies above the line connecting the two points $(n_i, v(n_i))$ and $(N, v(N))$. Thus the value $v(n_j)$ is greater than the linear combination of these two points at n_j.

Consider first the simple case of a linear network value curve and let that straight line be the line that connects points $(n_i, v(n_i))$ and $(N, v(N))$. As shown in Proposition 1(ii), the benefits of interconnection for network i are the same as for j. Since the value of interconnec-

Figure 1: Concave Network Value Function



tion for network i is area $A + B + C$, and the value of interconnection for j is the sum of areas $A + B + D + E$, we know that $C = D + E$.

Next consider the case of a concave network value curve shown in Figure 1. In this case, the value of interconnection for j is given by $A + D$, and the value of interconnection for i is the same, as in the linear case, or $A + B + C$. Since $B + C > D$, the value of interconnection is larger for network i than for network j. In the linear case, C represented the non-common value of interconnection for i, since A and B were shared in common with j, and $D + E$ represented the non-common value of interconnection for i. Concavity increases the non-common value of interconnection for the small network i and decreases the value for j.

When there are decreasing marginal returns to network size in the member's network value function, the small network will value interconnection more than the larger network. Assuming a bilateral Nash bargaining solution, the large networks will then charge the smaller ones for interconnection. In Figure 1, the amount paid will be $(B + C - D)/2$. In contrast, when there are increasing marginal returns to network size, the large network will value interconnection more than the small one. As a result, the small network will then charge the large one for interconnection.

This result extends to environments with more than two networks. To do so, we must make an assumption about the default network size for each network in the absence of the current contract. This model assumes that networks simultaneously negotiate interconnection with a common expected size of their ultimate network, namely the total number of network members N. As long as there is a common expected value of network size assuming interconnection, when network i is negotiating with network j, its default value of interconnection is then $n_i v(N - n_j)$ and its interconnected value is $n_i v(N)$. In equilibrium, the expectation of full interconnection is met. The proof of Proposition 1 then carries through in a manner comparable to the two network case above.

Generally, the concavity or convexity of the consumer's network value function is more critical for outcomes than the strength of the network effect itself. The slope of the value function $v'(n)$ measures the strength of the network effect. We see that the slope per se is not relevant to determining the sign of the net difference in values of interconnection; rather, the changes in slope, or concavity and convexity, are critical to determining the sign of the net differences. Strong network effects can be present without providing any indication of whether small networks will pay large networks for interconnection or not. This contrasts with Crémer et al. (2000) where the model yields concavity by construction so that, as the network effect becomes more pronounced, the larger network worsens its treatment of the smaller network.

When there are decreasing marginal returns to network size, small networks will prefer to band together and achieve their connections with large networks through intermediaries rather than through direct contracting. With decreasing marginal returns, the per member value of interconnection is lower for larger group sizes. Direct interconnections are more costly per member, assuming that the cost of forming an intermediary is moderate. That is, the valuation difference for the group will be less than the sum of the valuation differences of the individual members. As long as the costs of coalition formation are not too high, hierarchical contracting may be preferred to direct contracting with large providers.

This finding is consistent with the hierarchical nature of the Internet (see US Department of Justice (2000).) Smaller networks frequently purchase service from medium-sized networks who, in turn, purchase service from large networks. This hierarchy makes sense in an environment of decreasing marginal returns.

Under increasing marginal returns, a hierarchy does not make sense, since small size is then an advantage for bargaining. All else equal, large firms in fact prefer to break themselves up into smaller units to reduce their costs of interconnecting.

The role of mergers in affecting the values of interconnection is considered next.

4 Merger

Understanding the nature of interconnection value is important for policymakers trying to decide how to treat interconnection. Apart from regulating the nature of interconnection itself, for example, by requiring symmetric access fees,[16] there may also be regulatory questions with regard to mergers of interconnecting networks.[17] This section shows that the impact of mergers depends to a great extent on whether the value of interconnection has increasing or decreasing marginal benefits in the relevant range.

Network mergers may impact the actual costs of network provision through economies of scale or scope. A larger network may have lower physical costs per network user, thus increasing the efficiency of the merged networks while placing competing networks at a cost disadvantage. The ultimate dynamic effects of such cost efficiencies can be complex and are not considered here. Because we assume constant costs per subscriber, there are no cost effects of mergers in the model we develop. Rather, merger effects arise from changes in bargaining threat points.

Let there be three networks, i, j, and k, with n_i, n_j, and n_k members respectively. $N = n_i + n_j + n_k$. A merger between networks j and k, yields a new network of size $n_j + n_k$. Prior to the merger, each of the three networks must sign two different contracts in order to obtain interconnection, yielding a total of three contracts. After a merger, there is only one contract to sign since there are only two networks.

[16] The FCC has required symmetric access charges for local access, for instance, while other countries such as New Zealand have chosen to eliminate regulation of access charges (see Laffont and Tirole (2000).)

[17] In the WorldCom-Sprint merger, two of the largest Internet backbone companies proposed to join their backbones which would have resulted in them having 56% of Internet traffic (see US Department of Justice (2000).

Let $V_x^y(n)$ represent the value to network x of interconnection with network y when the expected post-contracting network size of both networks is n. We leave out n when the expected accessible network size is clear from the context. We refer to the "value difference" for x and y as the difference between V_x^y and V_y^x. The "aggregate value difference" for x is the difference between V_x^y and V_y^x summed over all other networks. That is, when there are τ other networks besides network x the aggregate value difference for x is given by $\sum_{y=1}^{\tau}(V_x^y - V_y^x)$. The aggregate value difference is important because, under Nash bargaining, the amount of network x's payments or revenues from interconnection is one half of the aggregate value difference. In order to examine the impact of a merger between two other networks on x, we calculate the aggregate value difference for x prior to a merger and that after a merger. If the value difference increases after a merger, that means that network x benefits more from interconnection after a merger, and thus it will pay more to other networks (or receive less from them) than it did prior to the merger.

Assume Nash bargaining. Because there are always gains from trade, networks believe there will be complete interconnection and this expectation is fulfilled in equilibrium, as in Chipty and Snyder (1999). In the expectations model, networks already have or believe they will have network interconnections with other networks, and thus the key issue in a bilateral negotiation rests on the impact of not having the current contract. Order of contracting does not matter.

In the expectations equilibrium, a network might expect to maintain full interconnection, because there will always be gains from interconnection that could be split between the networks. If there is a merger, the merger will affect the base valuation of the non-merging networks, but will not affect the base valuations of the merging networks.

Proposition 2 *Suppose there are three networks, i, j, and k, of arbitrary size, in which each member has a value function such that* $v'(n) > 0$*. Suppose that networks j and k merge. In equilibrium: (i) If* $v''(n) < 0$*, the merger results in an increase in the value of interconnection to network i relative to the merged networks. (ii) If* $v''(n) > 0$*, the merger will result in a decrease in the value of interconnection to network i relative to the merged networks.*

Proof of (i). Prior to the merger, network i will contract separately with network j and network k. In each negotiation, network i assumes that it will be successful in negotiating with the other network. Similarly, j and k assume they will be successful in negotiating with each other. In the negotiation with network j, network i will have a value of interconnection $V_i^j - V_j^i$

$$n_i v(N) - n_i v(N - n_j) - [n_j v(N) - n_j v(N - n_i)]. \tag{5}$$

Similarly, in negotiating with k, network i will have a value of interconnection $V_i^k - V_k^i$

$$n_i v(N) - n_i v(N - n_k) - [n_k v(N) - n_k v(N - n_i)]. \tag{6}$$

The sum of equations (5) and (6) then represents the pre-merger aggregate value difference for network i,

$$V_i^j - V_j^i + V_i^k - V_k^i = n_i v(N) - n_i v(N - n_j) - [n_j v(N) - n_j v(N - n_i)] +$$

$$n_i v(N) - n_i v(N - n_k) - [n_k v(N) - n_k v(N - n_i)]. \tag{7}$$

The post-merger aggregate value difference for network i is:

$$V_i^{jk} - V_{jk}^i = n_i[v(N) - v(n_i)] - [(n_j + n_k)[v(N) - v(n_j + n_k)]. \tag{8}$$

The difference between the post-merger aggregate value difference (8) and the pre-merger aggregate value difference (7) for i simplifies to

$$v(n_i + n_j) + v(n_j + n_k) - v(N) - v(n_j). \tag{9}$$

If (9) is greater than zero, then we know that the value difference for i increases as a result of a merger between j and k. Under Nash bargaining, this would imply that network i increases its payments to the merging networks as a result of a merger and that the merging networks would improve their bargaining position with respect to network i by merging. In contrast, if (9) is less than zero, then we know that the value difference for i decreases as a result of a merger. Under Nash bargaining, this would imply that network i would decrease its payments to the merging networks as a result of a merger.

We know that $n_i < n_i + n_j < N$ and that $n_i < n_i + n_k < N$. The concavity of the value function indicates that $v(n_i + n_j)$ lies above the linear combination of $(n_i, v(n_i))$ and $(N, v(N))$ at $(n_i + n_j)$ and that $v(n_i + n_k)$ lies above the linear combination of $(n_i, v(n_i))$ and $(N, v(N))$ at $(n_i + n_k)$.

Substituting the values of the linear combination at $(n_i + n_j)$ and $(n_i + n_k)$ respectively, both

$$v(n_i + n_j) > \frac{n_k v(n_i) + n_j v(N)}{n_j + n_k} \tag{10}$$

and

$$v(n_i + n_k) > \frac{n_j v(n_i) + n_k v(N)}{n_j + n_k}. \tag{11}$$

Summing (10) and (11), we have

$$v(n_i + n_j) + v(n_j + n_k) > v(N) + v(n_j)$$

or

$$v(n_i + n_j) + v(n_j + n_k) - v(N) - v(n_j) > 0.$$

So the post-merger aggregate value difference for network i is greater than the pre-merger aggregate value difference. □

Proof of (ii). The proof applies the definition of convexity rather than concavity, but follows the same method as for the proof of (i). □

From the perspective of a non-merging network i, the number of users who cannot be reached in absence of the contract with the merged entity is larger than the number of users who cannot be reached in absence of a contract with either of the constituent merging firms. In contrast, from the perspective of the merged network and its constituent original firms, n_i users are unreachable in case no contract is signed with network i both before and after a merger. So the change in the net value of interconnection arises from changes in the non-merging network's value of interconnection, not the merging network's value of interconnection.

Proposition 2 is similar to Proposition 2 of Chipty and Snyder (1999) who examine a monopoly seller contracting with multiple independent buyers in a one-way network (see Economides and White (1994)). In contrast, we examine a situation in which each firm is a supplier to the others and in which the network value arises from the summing of many individual valuations. The difference arises because the Internet is a two-way, point-to-point network. Each network supplies access to its members and receives access to the other networks' members. Assuming decreasing marginal returns, the members of the smaller network clearly experience a larger marginal gain from interconnection than the members of the larger network. At the small network, fewer members receive a larger marginal gain while at the large network, the marginal gain from interconnection is smaller, but a larger number of members receive it.

Note that no assumption is made in Proposition 2 about firm size. If two small networks merge and there is a concave value function, they will receive an advantage in their bargaining posture. However, they may still be net payers of fees to a third larger network; they would simply pay less than before.

For tractability, the analysis here focuses on a scenario with three networks. However, the result also applies when there are more networks. What matters for the bilateral contracting decisions is the change in the value difference between the merging network and the other contracting party. Adding other networks to the model is equivalent to raising the size of each side's "base" network but does not alter the outcome, because, in equilibrium, each network expects to maintain or achieve complete contracting, so the base of users a network expects to reach expands from one's own users to all the other users apart from those in the contract under consideration.

An interesting implication of Proposition 2 is that when there are increasing marginal returns to network size, the merging networks have a higher valuation difference than prior to the merger. Assuming the change in valuation differences is reflected in prices, a merger would yield higher expenses for the merging networks. In order for such a merger to make sense, either efficiencies must counterbalance the increased network costs or the large network may plan not to interconnect.

5 Disconnection

After there are structural changes in an industry that result in changes in network sizes, such as mergers, the prior equilibrium interconnection structure may no longer be an equilibrium. In particular, individual networks may choose not to interconnect with each other. Similarly, if there is a change in regulation that impacts a network industry, new equilibria may arise in which there is disconnection. Some might argue, for instance, that the Internet has a financial structure that is largely inherited from its oversight by the National Science Foundation. Prior to 1995, the NSF managed the structure of the network and the conditions of interconnection. Under NSF supervision, networks did not charge each other for interconnection. However, the NSF ceased its oversight of the Internet backbone network in 1995. To a large extent, networks have maintained their contracts from this period and continue not to charge each other for interconnection. After the oversight ended in 1995, a number of small networks claimed that larger networks refused to contract with them on the same terms.

Disconnection could occur if at least one of two connected networks believes its profits will increase from disconnection. Such disconnection can occur when an individual network that disconnects from one or more others expects to capture new subscribers as a result of the disconnection. How might this capture occur? Imagine first that a network that is a disconnector leaves another network as a disconnectee. Disconnection could potentially lead the disconnectee network to offer so much less value to its subscribers that it could not charge a price that would cover its costs. Recall next that by the assumption of this model, members may switch networks when their own network ceases to operate. If the disconnector network then captures switching subscribers, it may profit from disconnection.

Why might there be switching? If a network ceases service, its members are able to seek out new providers. For their choice decision, what matters is the joining cost s_j. Provided that the price of another provider and the joining cost are below the value derived from network membership, members may switch networks after their own network ceases to operate.

We now wish to consider the potential disconnections that might occur after a change in industry structure such as a merger or after a deregulation of an industry that formerly required interconnection or prohibited payments between networks. We assume that prior to the change in structure, there is full interconnection of all networks. But after a structural change, firms re-evaluate their own interconnection structures. Interconnection requires the continued agreement of both networks. The decision of a network to disconnect from another network leaves that other network providing its members a less complete access to other points than it did before. We no longer require that a network be individually sustainable. That is, we relax the assumption that $v(n_i) > c, \forall i$.

Suppose that after a merger, there are three networks: the new, larger merged network m, network i, and network h. They have n_m, n_i, and n_h members respectively, where n_m represents less than half of all members but $n_m > n_i$ and $n_m > n_h$.[18] The merged network is the largest network. The total number of members of all networks, prior to disconnection, is N, where $N = n_i + n_m + n_h$.

There are three primary cases to consider: one in which all networks continue to operate after the disconnection (the survival scenario), one in which a network ceases to operate due to lost network value from fielding a smaller network and its members do not switch to other networks (the elimination scenario), and one in which a network ceases to operate due to lost network value from fielding a smaller

[18]Network h is introduced so that no individual network will have more than half of the total network members. The condition is that $n_m < \frac{n_m+n_i+n_h}{2}$.

network and its members switch to other networks (the recapture scenario).

5.1 Survival Scenario.

The merged network m disconnects from network i and remains connected to network h. All networks continue to operate after the disconnection. Networks i and m therefore survive the loss in quality from connecting their members to a smaller universe. The disconnecting networks charge their members less for the lower quality service and, to avoid switching costs, the members remain with their original network.

The merged network alters its value of interconnection with network h. Network m's value of interconnection with h will change from

$$V_m^h(N) = n_m v(N) - n_m v(N - n_h) \tag{12}$$

to

$$V_m^h(N - n_i) = n_m v(N - n_i) - n_m v(N - n_i - n_h). \tag{13}$$

From the perspective of a member of network h, all members of network i and m can still be reached. As a result, in network h's calculus of interconnection, h's value of interconnection with m remains unchanged at:

$$V_h^m = n_h v(N) - n_h v(N - n_i).$$

If the result of disconnection is simply to shrink the network size of m and i, disconnection will not be a profitable choice.

Proposition 3 *Suppose there are three networks m, i, and h with n_m, n_i, and n_h members respectively where $n_m > n_h > n_i$. Suppose also that $c < v(n_m)$, $c < v(n_h)$,and $c < v(n_i)$. Then if $v''(n) < 0$, no network will seek disconnection.*

Proof. Suppose that network m were to disconnect with network i. Both network m and network i will continue to operate and service their members after disconnection since their costs of operation are lower than the price they charge to their members.

With decreasing marginal returns to network size, network h's value of interconnection is higher than network m's. The disconnection from i increases m's value of interconnection with h, as is seen immediately from noting that, by concavity, the difference in (13) must be greater than the difference in (12). An increase in m's value of interconnection with h, reduces the value difference. Thus, not only will m eliminate all payments from i as a result of disconnection and reduce the amount it can charge its own members, disconnection will also reduce payments from h.

From network i's perspective, disconnection will help i by eliminating the payment to network m, but hurt i even more by reducing the value of membership in network i to its users by a greater amount. Network i will thus not seek to disconnect.□

In short, if network m disconnects from a network that will continue to operate after disconnection, then network m both eliminates one set of payments (from network i) and reduces another set (those from network h). The payments from network h necessarily fall under a decreasing marginal returns environment. They fall because as network m decreases the size of the network accessible to its members, its value from interconnection with h increases, while h's value of interconnection remains the same as before, given that h is still connected to both i and to m. Thus the difference between the value of interconnection for h and m is reduced, as is the payment m receives from h.

Next, consider a situation in which, instead of network i continuing to exist after network m disconnects with it, network i ceases to operate.

5.2 Elimination Scenario.

In this case, the small network responds to disconnection by ceasing to operate. The network joining cost s_j for its members is such that the former members of network i do not rejoin any other networks. This then amounts to a reduction in the size of the potential networking population from N to $N - n_i$.

When will network i cease to operate? In order for a network to exit the market, the loss of its connection with the largest network must leave its members with insufficient value to cover the costs of operation.

The interconnection fee received by network x from network y when network x and y expect connection to n users is represented by $f_x^y(n)$.[19]

The profit for network i after network m disconnects, is given by

$$n_i v(N - n_m) + f_i^h(N - n_m) - n_i c$$

The exit condition is that profits turn negative. That is, the value to a member of network i of the network without m minus the per-member connection fee that is paid to network m is less than the physical cost of serving a member of the network, or

$$v(N - n_m) + \frac{f_i^h(N - n_m)}{n_i} < c. \tag{14}$$

Given the reduced size of the aggregate network, the first result we find is that when the network value function is linear, network m

[19]Payments from network x to network y are negative and payments from network y to network x are positive.

will not profit from eliminating another network.

Proposition 4 *Suppose there are three networks m, i, and h with n_m, n_h, and n_i members respectively. Suppose that if network m disconnects from network i, network i would cease operation, and the members of network i would not seek out a new network. Then if $v'(n) = l$, l a constant, neither network m nor i will seek disconnection.*

Proof. Obvious.

The reason that Proposition 4 holds is that with a linear network value function, the benefits of interconnection are the same for each interconnecting firm, so there are no net payments. Disconnection then reduces the quality of the network offered to members. From the perspective of a member of network m, this is a quality reduction that results in a lower willingness to pay, and lower revenue to m from consumers. Network i meanwhile experiences a complete elimination of profits from being forced to cease existing. However, its customers do not find new networks. So there is no potential gain from switching customers for network m. Network m thus experiences a decline in revenue but no decline in costs.

More generally, we can conclude that given the value of the network is larger without the elimination of network i, there always exists some set of payments that will ensure that network m would not choose to disconnect from i. The set of payments that would prevent disconnection may not always be feasible, however (see Segal (1999).) The costs of contracting, the costs of complexity, the difficulty of observing infractions, and the costs of enforcement may lead networks to sign contracts governing purely bilateral behavior. [20] We may thus find

[20] Furthermore, competition laws may constrain the ability to sign contracts governing behavior towards third parties.

that a contract between two networks will only govern behavior that directly affects both the two parties to the contract and, in particular, that one network cannot pay another to maintain connections with a third party network.

In a pure bilateral contract framework, reducing the size of the potential network from N to $N - n_i$ may sometimes be desirable for network m. This might occur under decreasing marginal returns if: (a) the value of interconnection increases dramatically more for network h than it does for network m, so that network m receives increased interconnection payments from shrinking the size of the networked population, (b) the elimination of payments from network i to network m does not reduce m's revenues by a large amount, and (c) the amount m can charge its own members does not change very much between N and $N - n_i$. This effect requires a large shift in the slope of the value function over a fairly narrow space. If such a shift occurs, reducing the size of the total potential network may push network h back so that its value of interconnection with m increases dramatically.

Proposition 5 *Suppose there are three networks m, i, and h with n_m, n_h, and n_i members respectively. Suppose that if network m disconnects from network i, network i would cease operation, and the members of network i would not seek out a new network. If*

$$f_m^h(N - n_i) - [f_m^h(N) + f_m^i(N)] > V_m^i(N)$$

then network m would disconnect with i in order to shrink the size of the total network.

Proof. The aggregate value difference for network m prior to any disconnection is given by

$$V_m^i(N) - V_i^m(N) + V_m^h(N) - V_h^m(N). \tag{15}$$

The aggregate value difference for network m after disconnecting and eliminating network i is given by m's value of interconnection with network h.

$$V_m^h(N - n_i) - V_h^m(N - n_i). \tag{16}$$

With a Nash equilibrium bargaining solution, network m will expect to keep half of the value difference. The change in fees paid to network m arising from disconnecting with i is given by (16) minus (15) and is

$$\frac{1}{2}(V_m^h(N - n_i) - V_h^m(N - n_i) - [V_m^i(N) - V_i^m(N) + V_m^h(N) - V_h^m(N)])$$

or

$$f_m^h(N - n_i) - [f_m^h(N) + f_m^i(N)].$$

Network m clearly experiences a decline in the value of its network to its own members, and hence the amount that it can charge its members, falls by

$$n_m[v(N) - v(N - n_i)],$$

which is simply the value of interconnection between m and i, $V_m^i(N)$.

Provided that the change in fees is positive and exceeds the loss from reduced charges to its own members, network m will find it more profitable to operate with a smaller total network size of $(N - n_i)$ rather than N. □

Disconnection is thus made plausible by the fact that, holding n_m and n_h constant, the difference in values of interconnection increases as N falls when $v(.)$ is concave. Henceforth, we assume concavity is not so extreme as to satisfy the condition of Proposition 5.

5.3 Recapture Scenario.

In this case, the small network responds to disconnection by ceasing to operate. Its members then migrate to other networks.

This will occur if the disconnector network believes (a) that ceasing to interconnect would lead other networks to exit, (b) that the disconnector network would succeed in recruiting at least a portion of the users of exiting networks, and (c) that, conditional on (a) and (b), at least one successful recruiting price yields higher profits to the disconnector network than with connection.

Disconnection presents an extreme case of quality degradation. Crémer et al. (2000) discuss the less extreme case of reducing the quality of connection between one network and another, resulting in increasing preferences for the larger network that experiences a lower level of quality loss than the smaller network.[21] Generally, networks will be most inclined to disconnect if they believe the disconnection will not shrink the number of users of networks. Cost and switching cost conditions must be met in order for disconnection to be profitable.

A network m may choose between partial disconnection (disconnecting from one or more networks) and complete disconnection (disconnecting from all other networks.) Before considering these cases, consider the three conditions that must be met in any instance of disconnection: the exit condition, the recruitment condition, and the profit condition.

5.3.1 Exit Condition

In order for consumers to switch, their network must cease to exist, otherwise they prefer to remain with their current network. Suppose that network m chooses to disconnect from network i. The exit condition for network i is then, as stated in (14),

[21] Crémer et al. (2000) do not focus on total disconnection as a possibility for the Internet. Service non-connection may be increasingly plausible as new advanced services are offered that rest on the Internet infrastructure. One such advanced service might be video conferencing, for example.

$$v(N - n_m) + \frac{f_i^h(N - n_m)}{n_i} < c.$$

5.3.2 Recruitment Condition

Once the exit condition has been satisfied, network m will seek to recoup some of the lost users, otherwise it loses the value of the connection to that group. Disconnection is more likely to be profitable when it leads members of the disconnected network to migrate from their current network to the disconnector's network.

In order for network m to have a chance of recruiting the former members of network i, network m must make the most attractive offer of any network to these members, that is

$$v_m - p_m > v_h - p_h$$

Furthermore, network m's offer must provide real value to potential members, or

$$v_m - p_m - s_j > 0.$$

Network m will be able to offer the lowest quality-adjusted price when network m gains the most from new subscribers. The price offered will depend on the profitable price of attracting new subscribers for network h. Generally, one role of a merger may be to increase the minimum quality-adjusted price offered by alternate networks. For example, if the largest firm were to merge with the firm that would have made the lowest offer among the alternate networks, the price the merged network could charge for recruiting new members would increase to the second lowest alternate price, a higher price than that charged prior to a merger.[22]

Proposition 6 *Suppose there are two networks m and h, of size n_m and n_h respectively where $n_m > n_h$. (i) If $v''(n) > 0$ and $v'(n) > 0$ then a new customer adds more value to the larger firm. (ii) If $v''(n) < 0$, $v'(n) > 0$, and*

$$\frac{v'(n_m)}{v'(n_h)} > \frac{n_h}{n_m}, \tag{17}$$

then a new customer adds more value to the larger firm.

Proof of (i). For a given value function $v(n)$, where $v'(n) > 0$, the value of network m is $n_m v(n_m)$. The value of a marginal customer for network m is $n_m v'(n_m) + v(n_m)$. Similarly, the value of a marginal customer for network h is $n_h v'(n_h) + v(n_h)$. Thus we see that if

$$n_m v'(n_m) + v(n_m) > n_h v'(n_h) + v(n_h)$$

network m derives a greater value from a new customer than network h. Consider that: (a) By assumption, $n_m > n_h$. (b) Given $v'(n) > 0$, $v(n_m) > v(n_h)$. (c) Given that $v''(n) > 0$, we know that $v'(n_m) > v'(n_h)$. It follows that $n_m v'(n_m) > n_h v'(n_h)$ and $v(n_m) > v(n_h)$. Adding these last two inequalities together, we have

$$n_m v'(n_m) + v(n_m) > n_h v'(n_h) + v(n_h).$$

Proof of (ii). We will meet the condition

$$n_m v'(n_m) + v(n_m) > n_h v'(n_h) + v(n_h)$$

when

$$n_m v'(n_m) - n_h v'(n_h) > v(n_h) - v(n_m).$$

[22]We assume that networks can offer special deals to new subscribers that are not available to existing subscribers. Such introductory offers are common in network industries.

The right hand side of this expression will always be negative, since $v'(n) > 0$ and $n_m > n_h$. Thus if the left hand side is greater than zero, as it is when

$$\frac{v'(n_m)}{v'(n_h)} > \frac{n_h}{n_m},$$

the condition will be satisfied. □

Here we find that small networks will generally have a lower value for a new customer than larger networks. This is always the case when there are increasing marginal returns to network size. Under decreasing marginal returns to network size when the value function is close to a linear function, the large network will value a new customer more than the small network. The linear case, for example, satisfies the condition and satisfies it by an amount strictly greater than 0. A slight movement from the linear case will not change the outcome.

In contrast, when there are extreme changes in the value curve between the small network size and the large network size, it is possible that the small network may benefit the most from a new customer. The intuition for this is that if there is an extremely large change in the value curve in the range at the small network's size, and no change at the large network's size, the large network derives no marginal benefit for its pre-existing subscribers from adding a new member, but the small firm does derive a considerable benefit for its subscribers from adding a new member. It is these benefits for pre-existing members that might overwhelm the simple effect of the increased value arising from adding one more subscriber.

Now consider the situation when there are multiple migrating consumers and there is declining marginal value of members.

Corollary 7 *Let there be a migrating class of customers of size n_i whose members individually join network h or network m where $n_m > n_h$. If $v''(n) < 0$, $v'(n) > 0$, and*

$$\frac{v'(n_m + n_i)}{v'(n_h)} > \frac{n_h}{n_m}, \tag{18}$$

then each migrating customer adds more value to the larger network.

Proof. Clearly, with a concave value function, the value of the left hand side is lower than at other values of the numerator in the domain from n_m to $n_m + n_i$. If the condition is met at the largest value of migrating consumers n_i, it will be met for all the prior values as well.□

5.3.3 Profit Condition

Finally, in order for network m to be willing to disconnect from network i, once the former members have moved to network m, the profits for network m must be greater under the disconnection scenario than under the connection scenario. The profits under disconnection are given by

$$\pi_m = n_m v(N) + n_i p_m + f_m^h(N - n_i) - (n_m + n_i)c$$

and the profits with interconnection are given by

$$\pi_m = n_m v(N) + f_m^i(N) + f_m^h(N) - n_m c$$

so the requirement that disconnected profits be greater than interconnected profits simplifies to

$$n_i p_m + f_m^h(N - n_i) - f_m^i(N) - f_m^h(N) - n_i c > 0.$$

5.3.4 Partial Disconnection

Network m may consider partial or complete disconnection in the context of migrating users. Consider the case of partial disconnection first.

Proposition 7 *Let there be three networks m, h, and i with* n_m*,* n_h*, and* n_i *members respectively. Assume that* $n_m > n_h > n_i$*. If*
(a) $v''(n) < 0$, *(diminishing marginal returns)*
(b) $v(N - n_m) + \frac{f_i^h(N-n_m)}{n_i} < c$, *(exit condition)*
(c) $\frac{v'(n_m+n_i)}{v'(n_h)} > \frac{n_h}{n_m}$ *(recruitment condition) and*
(d) $n_i(p_m - c) > f_m^i(N) + f_m^h(N) - f_m^h(N - n_i)$, *(profit condition)*
then network m will prefer disconnecting network i to full connectivity.

Proof. These conditions are that under decreasing marginal returns to network size, if the exit condition, the recruitment condition, and the profit condition are all satisfied, then network m will choose to disconnect from i. These conditions imply that network i will choose to exit after disconnection, that network m will succeed in recruiting the former members of network i, and that network m will receive higher increases in profits from recruiting new members than network h. As a result, network m will offer lower prices to switching users than network h, so network m will succeed in recruiting the switchers. Provided that a switcher generates higher profits for network m when the switcher is with network m than when it was with an independent company, network m will find partial disconnection profitable.

Interestingly, under decreasing marginal returns, if the physical cost of serving customers is significant, a large network may prefer to extract fees from another network's users via the user's network rather than direct customer fees. The reason is that adding new members requires paying the cost c for each member, whereas charging fees to the other network is based on a value analysis that, at the margin, ignores those fees.

5.3.5 Complete Disconnection

Partial disconnection is not the only scenario to consider. Under complete disconnection with migration, a network disconnects from all other networks.

When network m completely disconnects from other networks, network m neither pays nor receives fees for interconnection, so the profit condition changes to

$$n_i p_m - f_m^i(N) - f_m^h(N) - n_i c > 0.$$

Under decreasing marginal returns, the term $f_m^i(N) + f_m^h(N)$ is positive while under increasing marginal returns, the largest network m pays fees to other networks, so the term $f_m^i(N) + f_m^h(N)$ is negative.

Proposition 8 *Let there be three networks m, h, and i with n_m, n_h, and n_i members respectively. Assume that $n_m > n_h > n_i$. If*
(a) $v''(n) < 0$, *(diminishing marginal returns)*
(b) $v(N - n_m) + \frac{f_i^h(N-n_m)}{n_i} < c$, *(exit condition)*
(c) $\frac{v'(n_m+n_i)}{v'(n_h)} > \frac{n_h}{n_m}$, *(recruitment condition) and*
(d) $n_i(p_m - c) > f_m^i(N) + f_m^h(N)$, *(profit condition)*
then network m will find complete disconnection preferable to full connectivity.

Proof. Under decreasing returns with

$$\frac{v'(n_m + n_i)}{v'(n_h)} > \frac{n_h}{n_m},$$

the marginal value of a new user will be greatest to the larger network m as opposed to the smaller network h. Network m will then offer lower prices p_m to switching users than network h, and network m will succeed in recruiting the moving customers. If the profit condition is satisfied, disconnection will be preferred to retaining full connectivity. □

6 Welfare Effects

The prior sections have considered the asymmetric nature of interconnection incentives, particularly with respect to mergers and disconnection. What are the welfare impacts that arise from the asymmetric value of interconnection? From the perspective of social welfare, the predicted asymmetry between interconnection prices is not necessarily harmful. The asymmetry may shift rents between networks without causing any members to lose their network membership and without causing any networks to reduce the breadth of their interconnection.

In the model of this paper, the only welfare effect arises from disconnection. If network i ceases to exist as a result of a price increase or an explicit disconnection by network m, its members will then experience an exit cost s_e. If the members of network i are recruited to another network, which can happen if the joining cost satisfies $s_j + p_m < v(N)$, the welfare impact is based purely on switching costs. Each member of network i pays the switching costs $s_e + s_j$. These costs amount to $n_i(s_e + s_j)$.

If the network value function experiences extreme changes in slope, as in Proposition 5, network m may choose to disconnect from network i with the expectation that network i will then cease to exist and its members will not join any other network. The consumers of network i then lose all network benefits and have to pay an exit cost. Outside consumers experience a decline in welfare from losing connections to the members of network i. The welfare loss is then:

$$n_i[v(N) + s_e] + n_{-i}[v(N) - v(N - n_i)].$$

These potential welfare losses from disconnection are largely driven by the existence of the net payment of fees from one network to another. Clearly, in the absence of net payments, disconnection to achieve shrinkage would not occur, since the motivation for such shrinkage would be to improve the bargaining position of the large network with respect to the surviving networks. Disconnection might still occur in an attempt by a large network to recruit members. However, at least in a partial disconnection case, a large network will then be in no stronger a position to recruit members than a surviving small network, since the sole benefit of recruitment would be the increased direct revenue from new members, not the enhanced bargaining position from greater size.

7 Conclusion

This paper provides a foundation for evaluating the value of interconnection to networks of different sizes. Both network size and the shape of the representative consumer's network value function are critical to determining an individual network's assessment of the gains from interconnection. The model examines networks of fixed size with simple cost structures in order to focus on the issue of the value of interconnection. This model is thus an analysis of the incentives governing networks with mature installed bases. This contrasts with models based on unaffiliated consumer movement, such as Crémer et al. (2000) or Katz and Shapiro (1985). The model here may be most relevant in industries for which the installed base matters more than unaffiliated consumers.

When an individual network value function exhibits *declining* marginal returns to network size, a small network derives more value from interconnection than a large network. If this value difference extends to pricing, small networks may pay larger networks for interconnection. To receive more advantageous terms from larger networks, small networks may join together to form coalitions that lower the per-member fee for interconnection. A merger disadvantages the non-merging networks by increasing the loss in network size from failing to contract. However, mergers need not generate a loss in welfare from within this model, since the model focuses on an installed base of consumers with unit consumption.

In contrast, when the individual network value function exhibits *increasing* marginal returns to network size, a large network derives more value from interconnection than a small network. If this value difference extends to pricing, large networks may actually pay smaller networks for the privilege of interconnection. A merger would lead the merging networks to pay even more for interconnection.

The shape of the value function is more important for these results than its slope. That is, the direction of payment for interconnection does not depend on the actual strength of network effects in a given market but instead on the shape of the network value function.

Surprisingly, for some network value functions, a large network may prefer a smaller aggregate network size. Despite the increasing value of networks when they connect more users, a large network may improve its bargaining position when aggregate network size shrinks, because then remaining networks may experience larger values of interconnection with that network than they do when there are more users reachable on the aggregate network. As a result, a large network may disconnect from a smaller network when that disconnection will drive the smaller network and its members off the market, thus reducing the aggregate network size.

Note that some incentives for disconnection are driven by the fact that one network may have to pay another for interconnection. When there is no net payment between networks for interconnection, these incentives for disconnection are eliminated. Since reciprocal compensation and non-compensation rules common in many regulatory settings have the effect of eliminating payments between networks, the rules may reduce the incentive to disconnect networks. Such rules may be justified given that disconnection imposes social costs.

A key practical question is whether a consumer's value function can be well-represented and, if it can, whether the function has increasing or decreasing marginal returns in the region of interest. Many features of the Internet are consistent with a declining marginal returns value function and inconsistent with an increasing marginal returns function. Large Internet providers often charge smaller providers for interconnection, equal-sized providers do not charge each other for interconnection, and small providers often choose to contract with larger providers through a hierarchical structure rather than a direct contract. If we then conclude that Internet providers operate in a decreasing marginal returns environment, a merger of large providers is likely to yield a price increase to existing paying customers and may yield a conversion of non-paying customers to paying customers, as the size disparity rises.

Beyond the Internet backbone, we find instances both of decreasing marginal returns to network size and of increasing marginal returns. On the one hand, for example, we expect decreasing marginal returns in peer-to-peer file sharing networks, where the objective of users might be to find specific songs stored on other users' computers. New peers add fewer and fewer new songs to the network as the network size increases, because as the network grows larger, most of the songs a peer brings to the network are duplicative. Thus the marginal value of additional users declines as the number of users increases.[23]

[23] See Asvanund, Clay, Krishman, and Smith (2001) for an exploration of the declining

On the other hand, for example, we expect increasing marginal returns to network size when networks are completed. Once the last unconnected person joins a network, it may be possible to significantly enhance the utility of members. In the case of email, for instance, when the last person a member wants to reach joins the email system, the member may experience a significant enhancement in the value of the network.[24]

Overall, the model of this paper is highly stylized. Yet the model does appear to capture many of the basic industry facts related to Internet backbone interconnection. Useful additions would include an explicitly dynamic analysis of network size over time, a reduction in switching costs so that the installed base of users is more likely to switch between existing networks, and an introduction of of heterogeneous consumer types.

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marginal benefit of adding new users to OpenNap networks, that operate much like Napster prior to its shutdown.

[24]For an elaboration of this argument, see (Crémer, 2000).

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